UNIVERSITÉ DE MONTRÉAL

CENTRE DE RECHERCHE SUR LES TRANSPORTS

CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

COMMON BUS LINES

by

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This research was supported by a grant from the Ford Motor Company of Canada for applied research in the Analysis and Control of Urban Transportation networks.

Université de Montréal
Publication # 180 - Département d'informatique
Publication # 10 - Centre de recherche sur les transports
(Revised November 1974)

Dépôt légal - 1er trimestre 1978

ABSTRACT

In large cities operating an important public mass transit network, we are faced with the problem of "common bus lines": some routes share common sections and the passenger must select the buses he will probably use. While most computer packages code the section as a unique link and disregard the precise assignment of passengers on the distinct routes, we give a solution to this operational assignment based on the "perceived" travel time. Formulated as an optimization problem within a probabilistic context, the optimal subset of routes "to be selected" is obtained. A more detailed treatment on uniform and exponential cases for the time of the first vehicle to arrive on a route concludes the paper.

RESUME

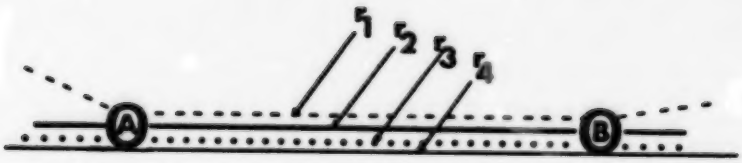
Dans les grandes villes ayant un service de transports publics important se pose le problème des lignes communes: plusieurs lignes empruntent parfois la même section sur leurs parcours. Alors que la plupart des systèmes actuels de planification informatisée codifient la section en question comme un seul lien parcouru par une seule ligne fictive et ignorent par conséquent l'affectation précise des passagers sur les lignes distinctes, nous proposons une solution à ce problème opérationnel basée sur la notion de temps "perçu". Formulé comme un problème d'optimisation dans un contexte probabiliste, le sous-ensemble optimal des lignes sélectionnées est alors obtenu. Une description plus détaillée pour les cas ou le temps d'arrivée du premier véhicule d'une ligne est distribué selon une loi uniforme et exponentielle complète cette étude.

In the operational planning of a public transit network, one is interested in loading the network in the most precise way regarding which routes will be used. The problem becomes more difficult in the case where one wants to take into account explicitly the routes which share a common section and one wants to calculate the loading of each of the particular sections which compose the trunk section.

1- INTRODUCTION

Suppose, as an example (figure 1), that a passenger wishing to travel from A to B has the choice of using one of the routes r_1 , r_2, r_3 or r_4 all of which go through A and B.

Figure 1


1
2
3
4
A
B

In most assignment techniques used to date, the link between A and B is coded as a single "ad hoc" fictitious link having such characteristics as to represent $r_1,\ldots,r_4$. For example, Dial [2] refers to such a link as a TLL (Trunk Line Link) of the form

$$TLL = A\ ,\ B\ ,\ D_L\ ,\ W_L$$

where L is a list of lines joining A and B , D_L and W_L are respectively the expected travel time and the expected waiting time in order to join A to B. It can be a major task for the planner to first determine L , D_L and W_L for each arc. Then the planner still has to evaluate the loading of each of the links as a separate problem.

The results we present here are part of our study on the MUCTC (Montreal Urban Community Transportation Commission) network. The network information has been defined in terms of lines and nodes, thus disregarding the usual definition in terms of arcs. Several procedures have been designed in order to deal with this structure. For example, one is able to select a node and obtain the set of routes passing through this node. Hence, the common routes will be detected easily (determination of L) .

The purpose of this paper is to present a probabilistic framework for the study of this problem which takes into account the frequencies and the speeds of each competing route. We claim that a passenger in A will not use just any of the routes $r_1,\ldots,r_4$. Because some routes might have a very long travel time to reach B, he may disregard them.

In that way, the passenger operates a selection: among the set of all routes passing through A and B, he will select a subset so as to minimize his total expected travel time which is made up of his expected waiting time plus his expected travel time. Once this subset is selected, the passenger will take the first vehicle serving one of the routes in this subset to arrive at A.

Lampkin and Saalmans [4] propose a procedure which does not include the minimization problem. Rather they note that

> "The problem of calculating T_{ij} (the total travel time) for journeys where more than one route runs from node i to node j was rather complicated. The definition of total travel time assumes that all passengers choose the route which would get them to their destination soonest, but unfortunately, this situation was difficult to analyse. What was done was to assume that passengers ignore obviously bad routes (for example, one for which the bus time is greater than the bus time + $\frac{1}{2}$ the inter-arrival time for the best route), and choose the first bus to arrive from one of the other routes."

They then go on to analyse the case of two or more common lines when the time for the first vehicle to arrive on each route is uniformly distributed. Expressions for the expected waiting time and for the probability of first arrival of a bus on a route are given. The expected total travel time can be readily calculated from these expressions. Silman, Barzily and Passy [6] have recently used the same approach.

In this paper we rather regard the problem of the determination of the expected travel time as an optimization problem. Our approach allows the consideration of any distribution function for first arrival time of a vehicle on a route. In practice, the cases of uniform and exponential distributions are of much interest: they are studied in greater detail.

2. THE EXPECTED TOTAL TRAVEL TIME

Let y_r be the time for the first vehicle to arrive on the route r where r is one of the routes between A and B . Let W_X be the waiting time for the first bus in the set of routes defined by X to arrive at A, going to B . $X = (x_1, x_2, \ldots)$ is a choice vector and is used to define a subset of acceptable routes as follows:

$$x_r = \begin{cases} 1 & \text{if the route } r \text{ is an element of the subset of acceptable routes} \\ 0 & \text{otherwise} \end{cases}$$

therefore

$$W_X = \min_{r:x_r=1} y_r \quad .$$

Let $p_r(z)$ and $P_r(z)$ be respectively the density and the cumulative distribution functions of the random variable y_r . Under the hypothesis that the y_r are independently distributed, the cumulative distribution

function $P_X(z)$ of W_X can be written as

$$\overline{P_X(z)} = \prod_r [\overline{P_r(z)}]^{x_r}$$

where $\overline{P(z)} = 1-P(z)$, as usual. W_X being a positive random variable, its expected value can be conveniently written as

$$E(W_X) = \int_0^\infty \overline{P_X(z)}dz \quad .$$

The hypothesis of independence of y_r is made to simplify the mathematics and rules out obviously any time-table.

We may want to visualize the situation as follows: each route defines a stationary point process of rather general probability structure and the processes for the different routes are statistically independent. A passenger then arrives at a random time. If $F_r(z)$ is the marginal headway distribution for the r-th route, then

$$P_r(z) = (1 - F_r(z)) \; / \int_0^\infty [1 - F_r(x)]dx \quad .$$

Given a selection vector X, the probability that a bus from route r passes first through A is

$$H_r(X) = Pr\,[y_r = \min_{i:x_i=1} y_i | X]$$

and

$$H_r(X) = \int_0^{\infty} x_r p_r(z) \prod_{i \neq r} [\overline{P_i(x)}]^{x_i} \, dz \quad .$$

If t_r denotes the in-vehicle travel time between A and B via route r, the expected total travel time T_X is given by

$$T_X = E(W_X) + \sum_r t_r H_r(X) \quad .$$

$E(W_X)$ is the expected waiting time and the second term on the right-hand side of T_X is the expected travel time.

We postulate that the expected total travel time T for a passenger going from A to B is

$$T = \min_X T_X$$

with the constraints

$$\sum_r x_r \geq 1 \qquad\qquad x_r = 0,1 \; .$$

We shall refer to this minimization problem as problem P and denote its solution by $\hat{X}$, then $T = T_{\hat{X}}$.

3- A HEURISTIC SOLUTION

When the number of common routes is not too large (less than eight or ten) the problem P can usually be solved by complete enumeration. Indeed T_X can be automatically evaluated for a given X with the aid of standard numerical integration techniques . In practice we have met very few cases of more than five or six common routes.

Another approach to the solution of problem P is to use the following heuristic algorithm.

> Sequence the routes in such a way that $t_1 < t_2 < \ldots$, and let $\overline{X} = (1,0,\ldots)$. $T_{\overline{X}} = E(y_1) + t_1$. Compute T_X for $X = (1,1,0,\ldots)$. If $T_X > T_{\overline{X}}$, stop, the heuristic solution is $\overline{X}$; otherwise, let $\overline{X} = X$ and compute T_X for $X = (1,1,1,\ldots)$. Continue this way until $T_X > T_{\overline{X}}$ or the vector $X = (1,1,\ldots,1)$ is attained in which case $\overline{X} = X$ is the heuristic solution.

We have not been able to characterize the class of problems for which the heuristic solution is the optimal one, nor been able to find a counter example. We conjecture that the heuristic produces the optimal solution. It is easily proven that this heuristic will produce the optimal solution whenever the times for the first arrival have identical distributions or the in-vehicle travel times t_r are equal. The motivation of the heuristic is quite simple: it is illogical for a passenger to let a bus of a given route go by and wait for a bus with longer in-vehicle time.

4- UNIFORM AND EXPONENTIAL DISTRIBUTION

Lampkin and Saalmans [4] considered the case where time y_r the time for the first vehicle to arrive on a given route r is uniformly distributed in the interval $(0,w_r)$. This situation occurs if headways on each route are constant, equal to w_r on route r, the starting time of each is uniformly distributed over w_r and the passenger arrives at an arbitrary time. The problem P for the determination of the expected total time is now given under the usual constraints as

$$T = \min_{X} \int_0^M \left\{ \prod_r (1-x_r z/w_r) + \sum_r \frac{t_r x_r}{w_r} \prod_{j\neq r} (1-x_j z/w_j) \right\} dz$$

where

$$M = \min_{i:x_i=1} w_i \quad .$$

The last expression generalizes the case treated by Lampkin and Saalmans. It is a simplification of analogous expressions in their paper.

It is instructive at this point to look at an example. We consider four routes going from A to B with parameters w_i and t_i as given in table 1.

TABLE 1

route	w_i	t_i
1	4.0	0.1
2	15.0	1.0
3	3.0	2.0
4	0.3	3.0

The optimal solution is to choose the routes 1 and 2 ($\hat{X} = (1,1,0,0)$). The expected travel time T equals 2.04. If the route 3 is also selected, X = (1,1,1,0),in addition to these routes, the expected travel time equals 2.32. If the route 1 alone is selected (X = (1,0,0,0)) the expected travel time equals 2.10 .

If the in-vehicle time for the first route is increased to 1.0 unit, the optimal solution is to choose the routes 1, 2 and 3 with an expected time of 2.64 . If the waiting interval of route 1 is modified to 9.0 (the other parameters being the ones of table 1) the optimal solution is again to choose the first three routes. The expected total travel time equals 2.89 . For each of these problems, the optimal solution would have been obtained with the heuristic algorithm.

As one may realize, the route selection procedure is sensitive to the in-vehicle travel time and waiting time.

A second important and interesting case is the one where the time y_r for the first vehicle to arrive on route r follows an exponential distribution

$$p_r(z) = \sigma_r^{-1} \exp(-z/\sigma_r) \quad .$$

This situation occurs when the bus arrivals are viewed as a Poisson process, where σ_r is the expected interarrival time on route r . If one uses the notation $F_r = \sigma_r^{-1}$ for the frequency of vehicles on route r , the problem P can be formulated under the usual constraints as

$$T = \min_{X} \; (1.0 + \sum t_r F_r x_r) \; / \; \sum_r F_r x_r \quad .$$

This problem belongs to the class of hyperbolic programming problems studied by Hammer and Rudeanu [3] and by Robillard [5] . It is easily shown that the heuristic algorithm produces in this case the optimal solution.

T measures the perceived time for a passenger going from A to B. As a function T is non-increasing with respect to the route frequencies and non-decreasing with respect to the in-vehicle time, as one would expect.

5- CONCLUSIONS AND COMMENTS

The objective of this paper has been to suggest a probabilistic framework for the study of common bus routes in the context of public transit.

As suggested by G. Newell in a private communication, the problem treated here produces a sub-optimal strategy. Indeed a "clever" passenger would observe both the route r of the arriving bus and how long he had waited for it. On the basis of this information he would then compare t_r with the conditional minimum trip time T for not taking the first vehicle given r and that no vehicle of type j , $j \neq r$ had arrived up to that time. The optimal strategy is to choose r if t_r is less

than T. The latter problem is somewhat more difficult to solve except for the case of exponential headways where the times to next arrivals are independent of past history. In such a case the sub-optimal strategy suggested here is optimal for the latter problem.

The procedures suggested here have been implemented as part of an assignment package for public transit called TRANSCOM [1]. They were used on the network of the (1970) Montreal Urban Community Transportation Commission (MUCTC) which has over 115 bus routes, 3 subway lines, 850 nodes and more than 2500 links.

The procedures suggested here acted as diversion methods within the assignment and brought more realism to the final loading of the network.

Finally, this approach can easily be adapted to deal with other units such as the value of waiting time and in-vehicle time, for example.

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